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ABSTRACT

This paper analyzes three possible research designs using each of the four types of sums of squares in the Statistical Package for the Social Sciences (SPSS). When the design is balanced (i.e., each cell has the same number of observations), all of the SPSS types of sums of squares yield equivalent results (testable hypotheses and sums of squares) for testing the significance of analysis of variance models. When the design is unbalanced, (i.e., not all cells have the same number of observations), only Type III and Type IV sums of squares agree. In addition, only the hypotheses being tested under Type III and Type IV sums of squares are interpretable. If there are empty cells in the design, none of the types of sums of squares agree. Moreover, only the hypotheses being tested under Type IV sums of squares are interpretable. However, these hypotheses are dependent on the variability and pattern of the missing cells. Thus, any conclusions based on these sums of squares should be made with caution. (Contains 15 tables.) (Author/SLD)

Running head: SUMS OF SQUARES

Interpreting the Four Types of Sums of Squares in SPSS

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Paper presented at the annual meeting of the Mid-South Educational Research Association, Bowling Green, KY, November 16, 2000.

Abstract

The present paper analyzes three possible research designs using each of the four types of sums of squares in SPSS.

When the design is balanced, (i.e., each cell has the same number of observations), all of the SPSS types of sums of squares yield equivalent results (testable hypotheses and sums of squares) for testing the significance of the analysis of variance models.

When the design is unbalanced (i.e., not all cells have the same number of observations), only Type III and Type IV sums of squares agree. Additionally, only the hypotheses being tested under Type III and Type IV sums of squares are interpretable.

If there are empty cells in the design, none of the types of sums of squares agree. Moreover, only the hypotheses being tested under Type IV sums of squares are interpretable. However, these hypotheses are dependent on the variability and pattern of the missing cells. Thus, any conclusions made based on these sums of squares should be made with caution.

Interpreting the Four Types of Sums of Squares in SPSS

A typical analysis of variance (anova) table consists of, among other things, source, sums of squares, degrees of freedom, mean square, F-calculated, and p-calculated. Since each F-calculated is found by a ratio of mean squares (which are found by a ratio of the corresponding sums of squares to its degrees of freedom), researchers need to be very careful as to how the sums of squares are computed. These sums of squares are computed by SPSS differently depending on the design being analyzed. When the design is balanced (i.e., each cell has the same number of observations), all of the SPSS types of sums of squares will yield equivalent results for testing the significance of the anova models. However, when the design is unbalanced (i.e., not all cells have the same number of observations), as is often the case, the hypotheses being tested are dependent on the type of sums of squares being used (computed). Thus, decisions made based on tests of significance when the design is unbalanced may differ depending on the type of sums of squares being used.

Although the SPSS Base 9.0 User's Guide (p.264-65) indicates which type of sums of squares to use in any given situation, the uninformed researcher might not know just which method to use. Thus, the uninformed researcher might select the type of sums of squares which supports some preconceived idea or the default method. Since Type III sums of squares is the default in SPSS (SPSS Base 9.0 User's Guide, 1999, p. 265), most uninformed researchers use this type of sums of squares in their analyses, also by default.

In order to make more sound decisions, researchers are encouraged to thoroughly investigate their data before doing any analyses. For the purpose of deciding which sums of squares to use, the researcher might do a cross-tabulation of the two variables (factors) and decide if the design is balanced, unbalanced, or if there are any empty cells.

The purpose of this paper is to illustrate, using a heuristic data set, the effects of using Type I, II, III, and IV sums of squares on a (a) balanced design, (b) unbalanced design with no missing data, and (c) unbalanced design with missing data.

Generally, the two-factor analysis of variance model is written as

$$Y_{ijk} = \mu_{..} + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk},$$

where:

$\mu_{..}$ is a constant

α_i are constants subject to the restriction $\sum \alpha_i = 0$

B_j are constants subject to the restriction $\sum B_j = 0$

$(\alpha B)_{ij}$ are constants subject to the restrictions:

$$\sum_i (\alpha\beta)_{ij} = 0, j = 1,\ldots,b$$

$$\sum (\alpha\beta)_{ij} = 0, i = 1,\ldots,a$$

ε_{ijk} are independent $N(0, \sigma^2)$

$i = 1,\ldots, a; j = 1,\ldots, b; k = 1,\ldots n_{ij}$

(Neter, Kutner, Nachtsheim, and Wasserman, 1996, p. 816)

If the n_{ij}'s are equal for all i and j, the design is said to be balanced. If they are unequal, the design is to be unbalanced. If any of the n_{ij}'s are empty, the design is said to have missing data.

According to Hocking, Hackney, and Speed (1978), it is this model formulation what causes the sums of squares to disagree.

An equivalent version of the factor effects model is obtained when the *ab* treatments are regarded without explicitly considering the factorial structure of the study. Now, the analysis of variance model is expressed in terms of the cell means μ_{ij}:

$$Y_{ijk} = \mu_{ij} + \varepsilon_{ijk}$$

where:

μ_{ij} are parameters

ε_{ijk} are independent $N(0, \sigma^2)$

$i = 1, \ldots, a; j = 1, \ldots, b; k = 1, \ldots, n$

(Neter et. al., 1996, p. 814)

That is, each μ_{ij} represents the mean response for the ith level of factor A and the jth level of factor B. It is these means that are to be estimated and hypotheses about them tested.

The null hypotheses tested in two-way analysis of variance are, according to Hinkle, Wiersma, and Jurs (1994, p. 412)

for rows:

$$H_{o1} = \mu_1. = \mu_2. = \ldots = \mu_j.$$

for columns:

$$H_{o2} : \mu._1 = \mu._2 = \ldots = \mu._k$$

for interaction:

$$H_{o3} : \text{all } (\mu_{jk} - \mu_j. - \mu._k + \mu) = 0$$

In other words, the first hypothesis (H_{o1}) is testing whether the mean for level 1 of factor A (when averaged over the levels of factor B) is the same as the mean for level 2 of factor A (when averaged over the levels of factor B) as well as the mean for level 3 of factor A (when averaged over the levels of factor B). Similarly, the second hypothesis (H_{o2}) is testing whether the means for the three levels of factor B are equal to each other (when averaged over the three levels of factor A). The third hypothesis is testing for no interaction among the levels of factor A and factor B.

Suppose a researcher collects data on three teachers (X, Y, and Z) and three teaching methods (lecture, cooperative learning, and web-based instruction). Assume the researcher calls the teachers factor A and the teaching methods factor B. Consequently, there would be three levels for factor A and three levels for factor B. Thus, there are (3*3) nine cells in the design matrix, see Table 1. Notice that the data in Table 1 represents a balanced design. As the paper proceeds, cell entries will be deleted as to make the design unbalanced. Ultimately, enough entries will be deleted as to have a design with missing data.

Insert Table 1 About Here

The data collected by the researcher was intended to determine the effect of the three teachers on student performance on a standardized test when taught using different methods. Factor A has three levels (X, Y, and Z) and factor B has also three levels (lecture, cooperative learning, and web-based instruction). A representation of this design is presented in Table 2. The data in this table represent population means. In other words, μ_{11} represents the mean response for the Lecture method as used by teacher X. Similarly, μ_{33} represents the mean response for the Web-based method as used by teacher Z. The marginal means, $\mu_{i.}$ and $\mu_{.j}$, represent the means for method i when averaged over teachers and teacher j when averaged over methods, respectively.

Insert Table 2 About Here

To test for differences in students' performance among the teaching methods when averaged over the different teachers, the following hypothesis may be used:

H_o: $\mu_{1.} = \mu_{2.} = \mu_{3.}$

Similarly, to test for differences in students' performance among the teachers when averaged over the different teaching methods, the following hypothesis may be used:

H_o: $\mu_{.1} = \mu_{.2} = \mu_{.3}$

To test for no interaction among teachers and methods, the hypothesis of interest may be written as

H_o: $\mu_{11} - \mu_{21} - \mu_{31} = \mu_{12} - \mu_{22} - \mu_{32}$ and

$\mu_{11} - \mu_{21} - \mu_{31} = \mu_{13} - \mu_{23} - \mu_{33}$

That is, are the differences among teachers constant across the teaching methods?

Although the preceding hypotheses are the most commonly presented in analysis of variance tables, other hypotheses are possible. For example, suppose a researcher believes that the Cooperative-learning method, all by itself, is just as good as the other two methods combined. The hypothesis of interest in such a situation may be written as

H_o: $\mu_{2.} = (1/2)(\mu_{1.} + \mu_{3.})$

Just as SPSS allows the researcher to create his/her own hypotheses, it also allows the researcher to specify which type of sums of squares to use. When the design is balanced, all four types of sums of squares will agree. When the design (balanced or unbalanced) has no missing data, the Type III sums of squares is the most commonly used. Moreover, since

Type III sums of squares is the default in SPSS, most uninformed researchers use the Type III sums of squares, also by default.

In the presence of missing data, Type I, II, and III sums of squares rarely have any reasonable interpretation. However, under the GLM univariate model dialog box, the user may select Type IV sums of squares. Then, SPSS will produce an interpretable hypothesis in which the cell means are balanced. However, as pointed out by Freund (1980), the Type IV sums of squares are dependent on the availability and pattern of the missing cells. Consequently, "if factor levels are reordered, a different hypothesis may result" (Pendleton, Tress, and Bremer, 1986, p. 2793) and "hence different sums of squares and *F*-values" (Milliken and Johnson, 1992, p. 187). To alert researchers of this fact, SPSS prints a message, after the anova table, indicating that the Type IV testable hypothesis is not unique.

The Type I sums of squares "is also known as the hierarchical decomposition of the sum-of-squares method. Each term is adjusted for only the term that precedes it in the model" (SPSS Base 9.0 User's Guide, 1999, p. 264). That is, given that teacher and method are the factors and teacher by method their interaction, SPSS will compute the Type I sums of squares differently depending on which factor is entered first. That is, if the factors are entered as teacher followed by method, then SPSS will be testing the significance of teacher alone, the significance of method given that teacher is also in the model, and the significance of the teacher by method interaction given that teacher and method are also in the model. Alternatively, if the factors are entered as method followed by teacher, then SPSS will be testing the significance of method alone, the significance of teacher given that method is in the model, and the significance of the method by teacher interaction given that method and teacher are in the model. Regardless of the order in which the factors are entered, the test for interaction remains the same. Thus, the main effect hypotheses are dependent on the order in which the factors are entered.

Type II sums of squares is not dependent on the order in which the factors were entered. This procedure calculates the sums of squares for an effect in the model adjusted for all other effects that do not contain the effect being examined. For example, given that teacher and method are the factors, Type II sums of squares will test the effect of teacher given that method is in the model and the effect of method given that teacher is in the model.

The general forms of Type I hypotheses for the μ_{ij} (cell means) model (Milliken and Johnson, 1986, p. 148) are presented in Table 3. The Type II hypotheses for the μ_{ij} (cell means) model (Pendleton et. al., 1986, p. 2791) are presented in Table 4. Similarly, the Type III hypotheses for the μ_{ij} (cell means) model (Milliken and Johnson, 1986, p. 150) are presented in Table 5. If there are missing cells in the design, Type IV sums of squares may be used with caution.

Insert Tables 3, 4, and 5 About Here

Numerical Examples

To better illustrate the effects of using the four SPSS types of sums of squares (Type I, II, III, and IV) on the different designs (balanced, unbalanced and no missing data, and missing data), the heuristic data set on Table 1 will be used.

Notice that Table 1 represents a balanced design. However, as the paper proceeds, cell entries will be deleted as to make the design unbalanced. Ultimately, enough entries will be deleted as to have a design with missing data.

Example 1: Balanced design

In the best of all worlds, each teacher would have the same number of students in each of the teaching methods. Such is the presumed situation in Table 1. Thus, the design is said to be balanced. A teacher by method cross-tabulation corroborates this, see Table 6.

Insert Table 6 About Here

As mentioned before, when the design is balanced all types of sums of squares agree. The SPSS output for the two factor anova model with interaction is presented in Table 7. Moreover, since the design is balanced, each sums of square is testing the same hypotheses. Namely, for the teacher main effect

$$(1/3)(\mu_{11} + \mu_{12} + \mu_{13}) = (1/3)(\mu_{31} + \mu_{32} + \mu_{33}) \text{ and}$$

$$(1/3)(\mu_{21} + \mu_{22} + \mu_{23}) = (1/3)(\mu_{31} + \mu_{32} + \mu_{33})$$

In other words, is the mean for the lecture method (when averaged over the three teachers) equal to the mean of the web-based method (when averaged over the three teachers) and is the mean for the cooperative learning method (when averaged over the three teachers) equal to the mean of the web-based method (when averaged over the three teachers)?

Insert Table 7 About Here

The corresponding hypotheses being tested for the method main effect are:

$$(1/3)(\mu_{11} + \mu_{21} + \mu_{31}) = (1/3)(\mu_{13} + \mu_{23} + \mu_{33}) \text{ and}$$

$$(1/3)(\mu_{12} + \mu_{22} + \mu_{32}) = (1/3)(\mu_{13} + \mu_{23} + \mu_{33})$$

In other words, is the mean for teacher X (when averaged over the three methods) the same as the mean of teacher Z (when averaged over the three methods) and is the mean for teacher Y (when averaged over the three methods) the same as the mean for teacher Z (when averaged over the three methods)?

All four sums of squares tested the same teacher by method interaction hypotheses. Namely,

$\mu_{11} - \mu_{13} = \mu_{33} - \mu_{31}$,

$\mu_{12} - \mu_{13} = \mu_{33} - \mu_{32}$,

$\mu_{21} - \mu_{23} = \mu_{33} - \mu_{31}$, and

$\mu_{22} - \mu_{23} = \mu_{33} - \mu_{32}$

Example 2: Unbalanced design

In most practical situations, not all teachers will have the same number of students participating in each of the teaching methods. Thus, the design will be unbalanced.

To illustrate the effects of using the four types of sums of squares (Type I, II, III, and IV) on an unbalanced (no missing data) design, some cell entries have been removed from Table 1. In actual research situations, this would indicate that information on some students is not available. For example, some students did not participate in some teacher by method combination. The data for this example is presented in Table 8. Again, this is an unbalanced (no missing data) design. A teacher by method cross-tabulation corroborates this, see Table 9.

Insert Tables 8 and 9 About Here

The results of applying the different types of sums of squares to the unbalanced design are presented in Table 10. Notice that Type III agrees with Type IV sums of squares. However, this is only because there is no missing data.

Insert Table 10 About Here

The hypotheses being tested under the Type I sums of squares are, for teacher main effect:

$.333\mu_{11} + .500\mu_{12} + .167\mu_{13} = .400\mu_{31} + .200\mu_{32} + .400\mu_{33}$ and

$.167\mu_{21} + .333\mu_{22} + .500\mu_{23} = .400\mu_{31} + .200\mu_{32} + .400\mu_{33}$

The corresponding hypotheses for the method main effect are:

$.337\mu_{11} + .226\mu_{21} + .119\mu_{22} + .397\mu_{31} = .112\mu_{12} + .225\mu_{13} + .385\mu_{23} + .007\mu_{32}\ .390\mu_{33}$ and

$.359\mu_{12} + .035\mu_{21} + .424\mu_{22} + .079\mu_{31} + .217\mu_{32} = .114\mu_{11} + .245\mu_{13} + .459\mu_{23} + .296\mu_{33}$

When the data was analyzed using Type II sums of squares, the hypotheses for the teacher main effect were:

$.390\mu_{11} + .385\mu_{12} + .225\mu_{13} + .007\mu_{21} + .112\mu_{23} = .119\mu_{22} + .397\mu_{31} + .266\mu_{32} + .337\mu_{33}$ and

$.094\mu_{11} + .224\mu_{21} + .305\mu_{22} + .417\mu_{23} = .074\mu_{12} + .019\mu_{13} + .318\mu_{31} + .231\mu_{32} + .452\mu_{33}$

The corresponding hypotheses for the method main effect are:

$.337\mu_{11} + .266\mu_{21} + .119\mu_{22} + .397\mu_{31} = .112\mu_{12} + .225\mu_{13} + .385\mu_{23} + .007\mu_{32} + .390\mu_{33}$ and

$.359\mu_{12} + .035\mu_{21} + .424\mu_{22} + .079\mu_{31} + .217\mu_{32} = .114\mu_{11} + .245\mu_{13} + .459\mu_{23} + .296\mu_{33}$

Clearly, none of the above hypotheses are readily interpretable. However, analyzing the data set using Type III sums of squares does produce interpretable hypotheses. For example, the hypotheses for the teacher main effect under Type III sums of squares are:

$(1/3)(\mu_{11} + \mu_{12} + \mu_{13}) = (1/3)(\mu_{31} + \mu_{32} + \mu_{33})$ and

$(1/3)(\mu_{21} + \mu_{22} + \mu_{23}) = (1/3)(\mu_{31} + \mu_{32} + \mu_{33})$

The corresponding hypotheses for the method main effect are:

$(1/3)(\mu_{11} + \mu_{21} + \mu_{31}) = (1/3)(\mu_{13} + \mu_{23} + \mu_{33})$ and

$(1/3)(\mu_{12} + \mu_{22} + \mu_{32}) = (1/3)(\mu_{13} + \mu_{23} + \mu_{33})$

The hypotheses being tested under Type IV sums of squares are the same (only because there are no empty cells) as those tested under Type III. Thus, such hypotheses are not reproduced here. However, notice that these hypotheses (under Type III or Type IV) are interpretable.

All four sums of squares tested the same teacher by method interaction hypotheses. Namely,

$\mu_{11} - \mu_{13} - \mu_{31} + \mu_{33}$,

$\mu_{12} - \mu_{13} - \mu_{32} + \mu_{33}$,

$\mu_{21} - \mu_{23} - \mu_{31} + \mu_{33}$, and

$\mu_{22} - \mu_{23} - \mu_{32} + \mu_{33}$

Example 3: Missing data design

In actual research, not only is it common to have unbalanced designs, but it is also common to have missing data. In other words, not only do the number of observations per cell are different, but some cells may even be empty. In a situation where some of the cells are empty, the design is said to be missing data.

To illustrate the effects of using the four types of sums of squares (Type I, II, III, and IV) on a missing data design, the cell entries for teacher X and web-based instruction have been completely removed. This could have been caused by a variety of reasons. For example, no students wanted to take such a class with teacher X. The data for this example is presented in Table 11. Again, this is an unbalanced/missing data design. A teacher by method cross-tabulation corroborates this, see Table 12.

Insert Tables 11 and 12 About Here

The results of applying the different types of sums of squares to the missing data design are presented in Table 13. Notice that Type III no longer agrees with Type IV sums of squares. This is because there is an empty cell in the design.

Insert Table 13 About Here

The hypotheses being tested under the Type I sums of squares are, for the teacher main effect:

$.333\mu_{11} + .667\mu_{12} = .400\mu_{31} + .200\mu_{32} + .400\mu_{33}$ and

$.167\mu_{21} + .333\mu_{22} + .500\mu_{23} = .400\mu_{31} + .200\mu_{32} + .400\mu_{33}$

The corresponding hypotheses for the method main effect are:

$.167\mu_{11} + .333\mu_{21} + .167\mu_{22} + .500\mu_{31} = .167\mu_{12} + .500\mu_{23} + .500\mu_{33}$ and

$.155\mu_{12} + .048\mu_{21} + .560\mu_{22} + .107\mu_{31} + .286\mu_{32} = .155\mu_{11} + .607\mu_{23} + .393\mu_{33}$

When this data set was analyzed using Type II sums of squares, the hypotheses for the teacher main effect were:

$.417\mu_{11} + .583\mu_{12} + .250\mu_{23} = .250\mu_{22} + .417\mu_{31} + .333\mu_{32} + .250\mu_{33}$ and

$.059\mu_{11} + .238\mu_{21} + .298\mu_{22} + .464\mu_{23} = .059\mu_{12} + .298\mu_{31} + .238\mu_{32} + .464\mu_{33}$

The corresponding hypotheses for the method main effect are:

$.167\mu_{11} +.333\mu_{21} + .167\mu_{22} + .500\mu_{31} = .167\mu_{12} + .500\mu_{23} + .500\mu_{33}$ and

$.155\mu_{12} + .047\mu_{21} + .560\mu_{22} + .107\mu_{31} + .286\mu_{32} = .155\mu_{11} + .607\mu_{23} + .393\mu_{33}$

Analyzing the data set using Type III sums of squares, produced the following hypotheses for teacher main effect:

$.500\mu_{11} + .500\mu_{12} + .167\mu_{23} = .083\mu_{21} + .083\mu_{22} + .417\mu_{31} + .417\mu_{32} + .167\mu_{33}$ and

$(1/3)(\mu_{21} + \mu_{22} + \mu_{23}) = (1/3)(\mu_{31} + \mu_{32} + \mu_{33})$

Similarly, the corresponding hypotheses for the method main effect are:

$.167\mu_{11} + .417\mu_{21} + .083\mu_{22} + .417\mu_{31} + .083\mu_{32} = .167\mu_{12} + .500\mu_{23} + .500\mu_{33}$ and

$.167\mu_{12} + .083\mu_{21} + .417\mu_{22} + .083\mu_{31} + .417\mu_{32} = .167\mu_{11} + .500\mu_{23} + .500\mu_{33}$

Clearly, none of these hypotheses are readily interpretable. Not even those produced by Type III sums of squares. Again, this is because there is an empty cell in the design.

When the data set was analyzed using Type IV sums of squares, SPSS produced interpretable hypotheses. However, as mentioned before, these sums of squares are dependent on the availability and pattern of the empty cells. To alert researchers of this fact, SPSS displays a message after the anova table indicating that the Type IV testable hypothesis is not unique. Similar messages are displayed throughout the print out if the Contrast Coefficient Matrix, under the GLM Univariate Options, is requested.

The hypotheses being tested under the Type IV sums of squares are, for the teacher main effect:

$(1/2)(\mu_{11} + \mu_{12}) = (1/2)(\mu_{31} + \mu_{32})$ and

$(1/3)(\mu_{21} + \mu_{22} + \mu_{23}) = (1/3)(\mu_{31} + \mu_{32} + \mu_{33})$

The corresponding hypotheses for the method main effect are:

$(1/2)(\mu_{21} + \mu_{31}) = (1/2)(\mu_{23} + \mu_{33})$ and

$(1/2)(\mu_{22} + \mu_{32}) = (1/2)(\mu_{23} + \mu_{33})$

Again, although the hypotheses produced by the Type IV sums of squares are readily interpretable, if any conclusions are to be made based on these sums of squares, they should be made with caution.

All four sums of squares tested the same teacher by method interaction hypotheses. Namely,

$\mu_{11} + \mu_{32} = \mu_{12} + \mu_{31}$,

$\mu_{21} + \mu_{33} = \mu_{23} + \mu_{31}$, and

$\mu_{22} + \mu_{33} = \mu_{23} + \mu_{32}$

To illustrate the dependency of the Type IV sums of squares on the availability and pattern of the empty cells, the data in Table 11 was rearranged so that the cell for teacher Z and cooperative learning method would now be empty, see Table 14.

Insert Table 14 About Here

As expected, the sums of squares for each design changed as the location of the empty cell was changed, see Table 15. Similarly, the hypotheses being tested under the given type of sums of squares also changed. What did not change was the fact that only Type IV sums of squares produced interpretable hypotheses. Of course, any conclusions made based on these sums of squares should be made only with caution.

Insert Table 15 About Here

Each example was also analyzed using SAS. At each step, the results, testable hypotheses and sums of squares, were identical, within rounding errors, to those obtained by SPSS.

Conclusion

Through the paper, three possible research situations were presented and analyzed using each of the four types of sums of squares in SPSS.

The first situation involved all three teachers (X, Y, and Z) using all three teaching methods (lecture, cooperative learning, and web-based instruction). Additionally, it was assumed that there was an equal number of participants per teacher per teaching method. In other words, the design was assumed to be balanced. When the two factor anova model with interaction was executed using the different types of sums of squares, it was concluded that, as expected, all types of sums of squares agreed. Moreover, the hypotheses being tested were interpretable.

The second situation also involved all three teachers using all three teaching methods. However, not all teachers had the same number of participants per teaching method. Thus, the design was unbalanced. When the two factor anova model with interaction was executed using the different types of sums of squares, only Type III and Type IV sums of squares agreed (only because there were no empty cells). Additionally, only the hypotheses being tested under Type III and Type IV sums of squares were interpretable.

The third situation involved two teachers using all three teaching methods and one teacher using only two teaching methods. Thus, the design had an empty cell. Execution of the anova model with interaction showed that none of the types of sums of squares agreed. Additionally, only the hypotheses being tested under Type IV sums of squares were interpretable. However, as mentioned before, these sums of squares are dependent on the variability and pattern of the missing cells. Thus, if any conclusions are to be made based on these sums of squares, they should be made with caution.

In a research situation where the design is balanced, the researcher may use any of the four types of sums of squares and obtain comparable results. However, if the design is unbalanced (but has no empty cells), the researcher may use either Type III or Type IV sums of squares. In the case of empty cells, only Type IV sums of squares may be used, with caution.

Instead of allowing statistical packages to generate and test their own hypotheses, researchers are encouraged to create and test their own hypotheses. In doing so, the hypotheses might be more meaningful to the researchers.

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Table 1. Design matrix for balanced design

	Method		
Teacher	Lecture	Cooperative Learning	Web-based
X	7	7	8
	8	8	7
	7	9	7
Y	9	8	6
	7	9	8
	7	7	9
Z	7	7	8
	9	7	7
	7	7	7

Table 2. Cell means

	Method			
Teacher	Lecture	Cooperative Learning	Web-based	Marginal
X	μ_{11}	μ_{12}	μ_{13}	$\mu_{1.}$
Y	μ_{21}	μ_{22}	μ_{23}	$\mu_{2.}$
Z	μ_{31}	μ_{32}	μ_{33}	$\mu_{3.}$
Marginal	$\mu_{.1}$	$\mu_{.2}$	$\mu_{.3}$	$\mu_{..}$

Table 3. Type I hypotheses for cell means model

Source of variation	Type I hypothesis
A	$\frac{1}{n_{1\cdot}}\sum_j n_{1j}\mu_{1j} = \frac{1}{n_{2\cdot}}\sum_j n_{2j}\mu_{2j} = \ldots = \frac{1}{n_{a\cdot}}\sum_j n_{aj}\mu_{aj}$
B	$\sum_i \left(n_{ij} - \frac{n_{ij}^2}{n_{i\cdot}}\right)\mu_{ij} = \sum_{j'\neq j}\sum_i \frac{n_{ij}n_{ij'}}{n_{i\cdot}}\mu_{ij'}$ for j = 1, 2, …, b
A*B	$\mu_{ij} - \mu_{i'j} - \mu_{ij'} + \mu_{i'j'} = 0$ for all i, j, i', and j'

Table 3. Type II hypotheses for cell means model

Source of variation	Type II hypothesis
A	$\sum_j n_{ij}\mu_{ij} = \sum_{i'} \frac{1}{n_{\cdot j}} \sum_j n_{ij}n_{i'j}\mu_{i'j}$
B	$\sum_i \left(n_{ij} - \frac{n_{ij}^2}{n_{i\cdot}}\right)\mu_{ij} = \sum_{j'\neq j}\sum_i \frac{n_{ij}n_{ij'}}{n_{i\cdot}}\mu_{ij'}$ for j=1,2,…b
A*B	$\mu_{ij} - \mu_{i'j} - \mu_{ij'} + \mu_{i'j'} = 0$ for all i, j, i', and j'

Table 5. Type III hypotheses for cell means model

Source of variation	Type III Hypothesis
A	$\bar{\mu}_1. = \bar{\mu}_2. = \ldots = \bar{\mu}_a.$
B	$\bar{\mu}._1 = \bar{\mu}._2 = \ldots = \bar{\mu}._b$
A*B	$\mu_{ij} - \mu_{i'j} - \mu_{ij'} + \mu_{i'j'} = 0$ for all *i, j, i'*, and *j'*

Table 6. Teacher by Method cross-tabulation (balanced design)

	Method			
Teacher	Lecture	Cooperative Learning	Web-based	Total
X	3	3	3	9
Y	3	3	3	9
Z	3	3	3	9
Total	9	9	9	27

Table 7. Sums of squares for balanced design

	Type I	Type II	Type III	Type IV
Teacher	.889	.889	.889	.889
Method	.222	.222	.222	.222
Teacher*Method	1.556	1.556	1.556	1.556

Table 8. Design matrix for unbalanced design

	Method		
Teacher	Lecture	Cooperative Learning	Web-based
X	7 8	7 8 9	8
Y	9	8 9	6 8 9
Z	7 9	7	8 7

Table 9. Teacher by Method cross-tabulation (unbalanced design)

	Method			
Teacher	Lecture	Cooperative Learning	Web-based	Total
X	2	3	1	6
Y	1	2	3	6
Z	2	1	2	5
Total	5	6	6	17

Table 10. Sums of squares for unbalanced design

	Type I	Type II	Type III	Type IV
Teacher	.898	1.158	1.910	1.910
Method	.691	.691	.497	.497
Teacher*Method	2.009	2.009	2.009	2.009

Table 11. Design matrix for missing data design

	Method		
Teacher	Lecture	Cooperative Learning	Web-based
X	8	8 9	
Y	9	8 9	6 8 9
Z	7 9	7	8 7

Table 12. Teacher by Method cross-tabulation (missing data design)

	Method			Total
Teacher	Lecture	Cooperative Learning	Web-based	
X	1	2		3
Y	1	2	3	6
Z	2	1	2	5
Total	4	5	5	14

Table 13.

Sums of squares for missing data design

	Type I	Type II	Type III	Type IV
Teacher	1.300	1.232	1.883	1.941
Method	1.182	1.182	.963	1.491
Teacher*Method	1.351	1.351	1.351	1.351

Table 14. Design matrix for missing data design (rearranged)

	Method		
Teacher	Lecture	Cooperative Learning	Web-based
X	8	8 9	7
Y	9	8 9	6 8 9
Z	7 9		8 7

Table 15.

Sums of squares for missing data design (rearranged)

	Type I	Type II	Type III	Type IV
Teacher	.417	.484	.759	.972
Method	2.818	2.818	2.913	2.963
Teacher*Method	.599	.599	.599	.599

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